Adaptation of the fountain code for BI-AWGN channel with the use of the polar code for error correction

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***Abstract*—The following publication presents the extension of an erasure correction fountain code with an error correction polar code that allows for the efficient adaptation of a fountain code for transmission over the BI-AWGN channel. The fountain codes are rateless codes designed for erasure channels that allow for packets reconstruction after they have been erased during a transmission. Generally, a packet transmitted via the BI-AWGN channel is considered erased when it did not arrive at the receiver or when it contains errors resulting from the transmission medium noise. Since when more erasures occur, more additional packets need to be generated by the fountain coder, we propose to use additional error correction coding that decreases the number of erased packets. It is achieved due to increased packet redundancy and error-correction decoding at the receiver side. For this purpose, we have selected the polar code because of its low complexity and excellent performance. This paper describes the proposed coding scheme and the simulation results analysis for the case of communication via the BI-AWGN channel.**

Keywords—fountain code, Raptor code, polar code, erasure channel, BI-AWGN channel

I. Introduction

This research paper presents a proposition of a fountain code adaptation for a Binary Input Additive White Gaussian Noise (BI-AWGN) channel with the use of polar codes for error correction. Fountain codes are a class of network coding schemes that can provide efficient and robust communication over noisy channels [4]. The use of the polar code [4], a powerful error-correcting code, as the additional error correction method for the fountain code output symbols allows for easy adaptation of the erasure fountain code to BI-AWGN channel allowing at the same time a significant improvement in performance. The design and investigation of this fountain code for the BI-AWGN channel are important for the development of reliable and efficient communication systems. This paper aims to provide a comprehensive analysis of the performance of the proposed fountain code and to demonstrate its advantages over existing methods.

II. Fountain Code with the Usage of Polar Code

A. Raptor Codes

Raptor codes are a type of fountain code that are designed to provide efficient and reliable communication over lossy channels [4]. Fountain codes are a class of error-correcting codes that allow for the efficient encoding of a message into an arbitrarily large number of encoded packets, from which an arbitrary subset can be used to reconstruct the original message. This makes them well-suited for use in wireless communication systems, where packets may be lost due to fading, interference, or other distortions and noises.

The structure of Raptor codes is divided into two main parts (Fig. 1): the outer code and the LT encoder [4]. The LT encoder is responsible for generating the encoded packets providing basic erasure correction, while the inner code is responsible for additional error correction.

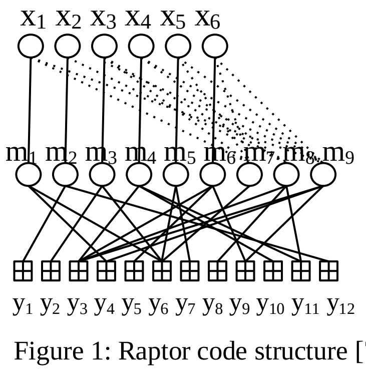

Figure 1: Raptor code structure [7].

The inner code typically consists of a channel code such as a low-density parity-check code or polar code [4]. These codes are used to correct errors that occur in the transmitted packets and recover the original data.

The LT encoder is responsible for generating the encoded packets. It uses a low-density generator matrix and a systematic encoding process to generate the encoded packets. The encoded packets contain the original data as well as additional redundant information added by the inner coder that is used for error correction.

Used coding scheme consists of initial encoding and LT encoding. The advantage of this approach is that instead of developing a more complicated rule for creating a single generation matrix, the entire process of encoding and decoding was broken into two separate steps [7]. As a result, we first map the k symbols of the informative $\boldsymbol{x}_{\mathrm{i}}$ into $\boldsymbol{m}$ intermediate symbols $\boldsymbol{m}_{\mathrm{j}}$. The LT encoder is then run to generate a fountain of output symbols.

The decoder is the combination of LT decoder and pre-coder. The decoding of Raptor codes is similar to that of LT codes [7]. Decoding starts in the reverse order of the coding process. In the first stage it uses LT decoder and then decode the intermediate code using the internal code decoder.

Let's further analyze the algorithm for encoding and decoding Raptor codes and their structure, which is shown in Fig. 1 [7].

We start coding by calculating the intermediate symbol vector $\boldsymbol{m}$ based on the assumed vector $\boldsymbol{x}$, using the formula (precoding):

$$\boldsymbol{m} = \boldsymbol{x}\boldsymbol{G}_m \quad (1)$$

We get a pre-code generating matrix Gm

The next step is to calculate the vector $\boldsymbol{y}$ using the intermediate symbol vector $\boldsymbol{m}$, using the formula (LT-coding):

$$\boldsymbol{y} = \boldsymbol{m}\boldsymbol{G}_{LT} \quad (2)$$

By substituting the matrix $\boldsymbol{G}_{LT}$ into equation (2), we obtain the following equations generating from which we calculate the individual symbols of codeword $\boldsymbol{y}$.

A detailed example of both encoding and decoding is included in the paper [7].

B. Packets erasure

One issue that can occur in the use of Raptor codes is packet erasure. Packet erasing happens when some of the encoded packets are lost during transmission, but the decoder is unable to reconstruct the original message due to the lack of enough redundant packets. This can happen when the number of lost packets is higher than the number of redundant symbols, leading to an inability of the decoder to recover the original message. To counter this, some schemes have been proposed that use multiple rounds of encoding and transmission in order to increase the number of redundant symbols, and hence the robustness of the system against packet loss [9].
Example of packets erasing is shown in equation (11).

$$G_{LT\_before\_erasing} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$G_{LT\_after\_erasing} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \quad (11)$$

C. BI-AWGN channel

A BI-AWGN (binary additive white Gaussian noise) channel [10] is a communication system in which a binary signal (consisting of two levels, such as 0 and 1) is transmitted through a channel that adds white Gaussian noise to the signal. White Gaussian noise is a type of random noise that is characterized by a normal distribution of values and a flat frequency spectrum. The noise is considered "white" because it has equal power at all frequencies. The additive aspect of the noise means that it is simply added to the signal, rather than being multiplied by it. The channel is typically modeled as a linear, time-invariant system, and the goal of communication over such a channel is to recover the original binary signal with as little error as possible.

D. Polar code

Polar codes are a class of error-correcting codes [11]. They are based on the concept of channel polarization, which refers to the phenomenon where a large number of independent and identical binary-input channels can be transformed into a smaller number of channels with different reliabilities. The polar codes are designed to take advantage of this phenomenon by allocating more code bits to the more reliable channels and fewer code bits to the less reliable channels.

Polar codes are particularly well-suited for use over binary symmetric channels (BSC) and binary erasure channels (BEC). BSC is a channel in which each bit of the transmitted signal is flipped with a certain probability, and BEC is a channel in which each bit of the transmitted signal is erased with a certain probability. Both channels introduce errors into the transmitted signal, but they do so in different ways.

For BSC, polar codes provide a significant coding gain over other codes such as Reed-Solomon codes [12], which leads to better performance in terms of the bit error rate for the same block length.

On the other hand, for BEC, polar codes can achieve the capacity of the channel with a relatively small block length [13], this means that polar codes are able to correct a large number of erasures with a small number of redundant bits.

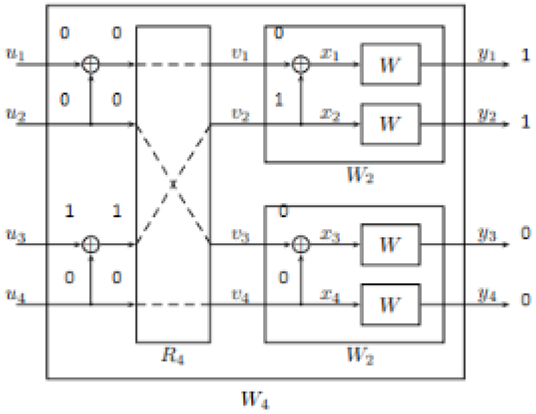

Figure 2: Encoder scheme.

III. MEASUREMENT PROCEDURE

For our investigation purposes, we've implemented a pipeline processing procedure. This procedure consists of six steps: decoder initialization, receiving of the incoming packet and the corresponding column of the G_{LT} matrix, packet and column processing, verification of the processed column, determining column weights and checking if the message was received correctly. If the message was received incorrectly the steps 2 through 6 are executed until, it will be determined otherwise.

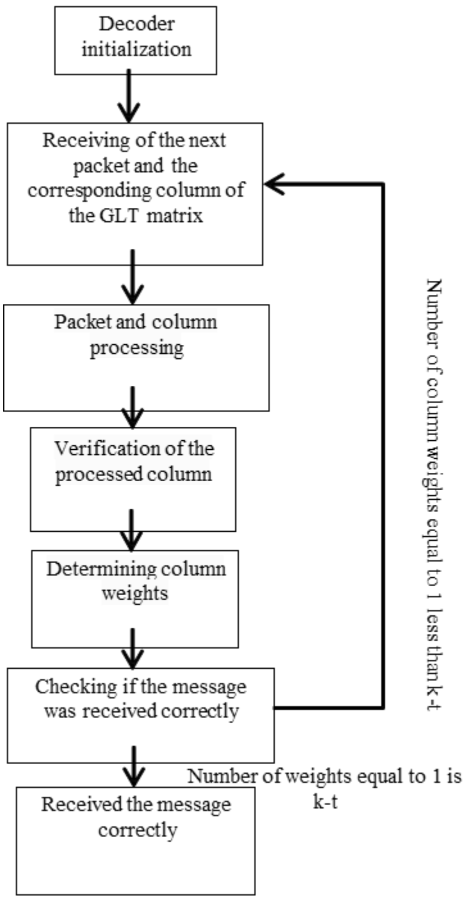

Figure 2: Procedure of pipeline processing.

The decoder initialization step sets the initial conditions for the pipeline processing procedure. In the next step, we receive an incoming packet and the corresponding column of the G_{LT} matrix. This is followed by package and column processing, where we perform various operations on the received packet and column. The processed column is then verified to ensure that it has been processed correctly. The column weights are then updated, which are used to check if the message was received correctly. If the message is not received correctly, the algorithm goes back to the second step, otherwise, it proceeds to the final step, where the message is declared as correctly received. This pipeline processing procedure allows us to efficiently and effectively investigate the performance of the fountain code for a BI-AWGN channel with the use of polar codes for error correction.

IV. RESULTS

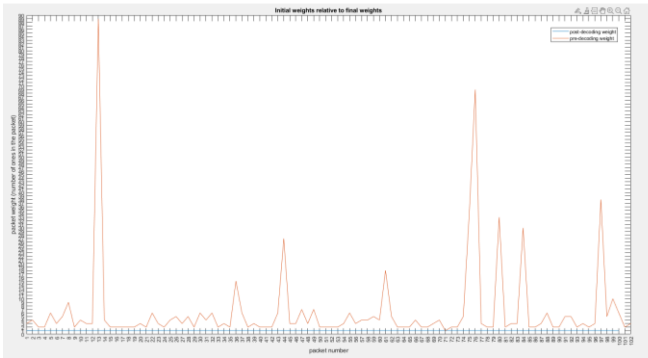

Figure 3: Initial weights relative to final weights. 100 packets with 12 bits length.

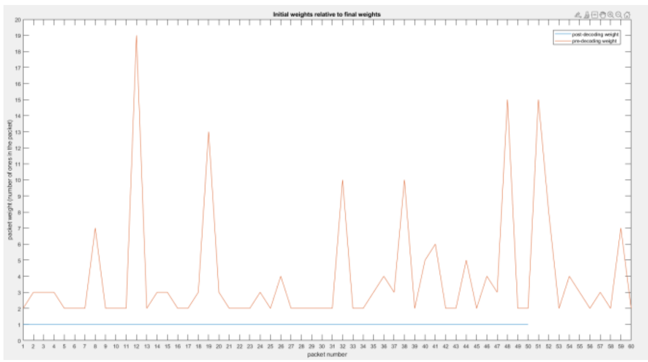

Figure 4: Initial weights relative to final weights. 50 packets with 50 bits length.

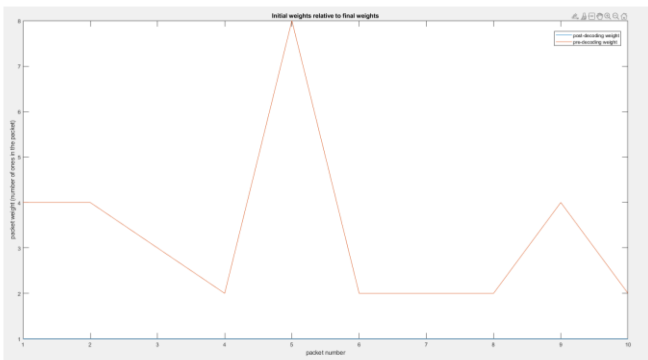

Figure 5: Initial weights relative to final weights. 10 packets with 40 bits length.

After successful decoding, only one 1 is left in each packet. With a larger number of packets, a reduced number of packets needed to send the same information may be noticed (redundancy has been removed).

V. Summary

In conclusion, the results of this study demonstrate that the proposed fountain code with the use of polar codes for error correction is a highly efficient and robust method for communication over BI-AWGN channels. These results have important implications for the development of reliable and efficient communication systems. The presence of a polarizing code confirms the increased efficiency of decoding transmitted packets.

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